

Capacitated Vehicle Routing Problem with Time Windows

W. David Fröhlingsdorf

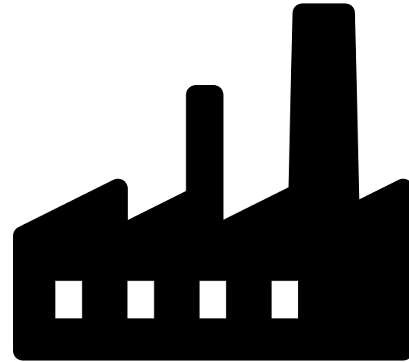


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**Capacitated
Vehicle
Routing
Problem *with*
Time
Windows**

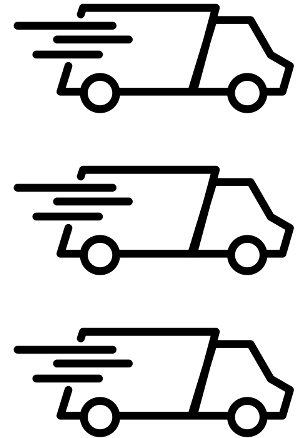
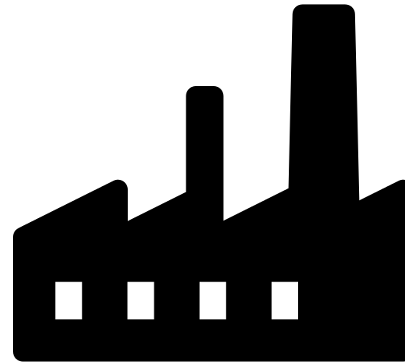
CVRPTW

Central Depot



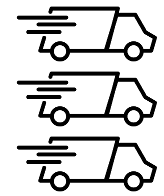
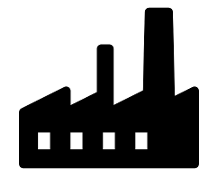
CVRPTW

Central Depot
Vehicle Fleet



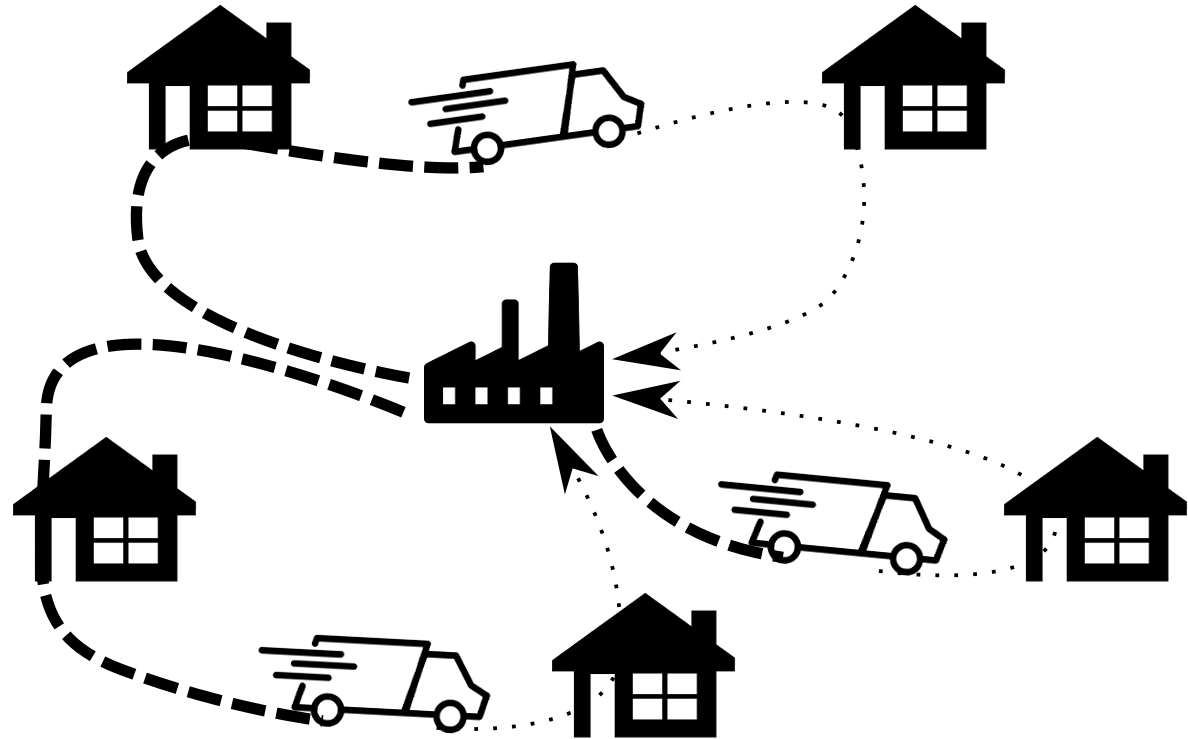
CVRPTW

Central Depot
Vehicle Fleet
Customers



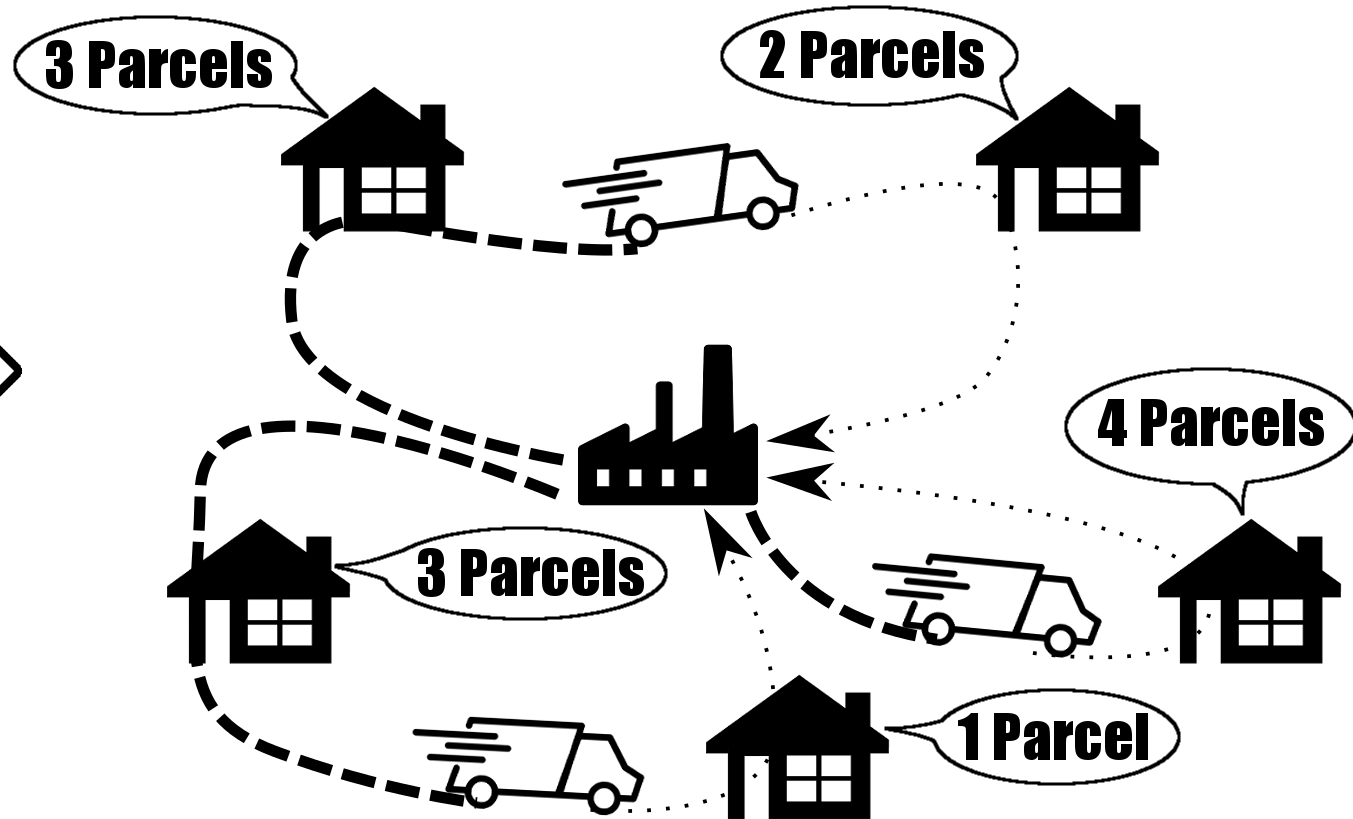
CVRPTW

Central Depot
Vehicle Fleet
Customers



CVRPTW

Central Depot
Vehicle Fleet
Customers
Demand



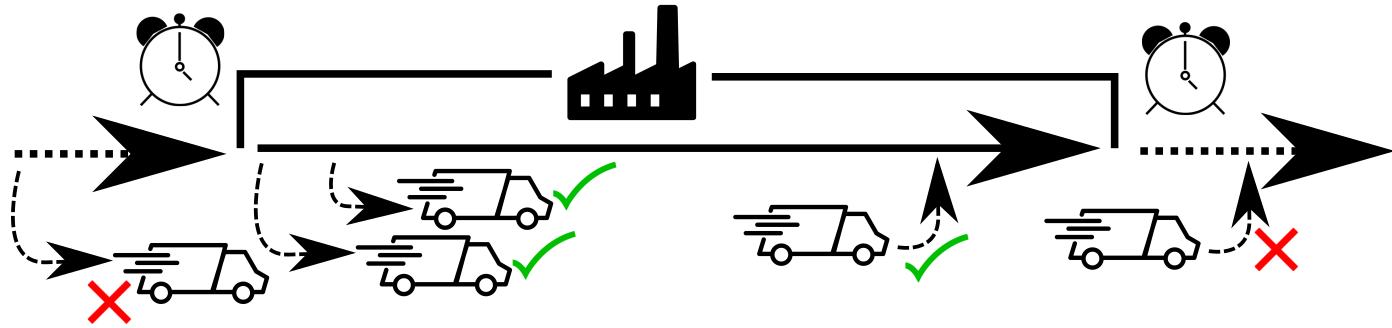
CVRPTW

Central Depot
Vehicle Fleet
Customers
Demand
Capacity



CVRPTW

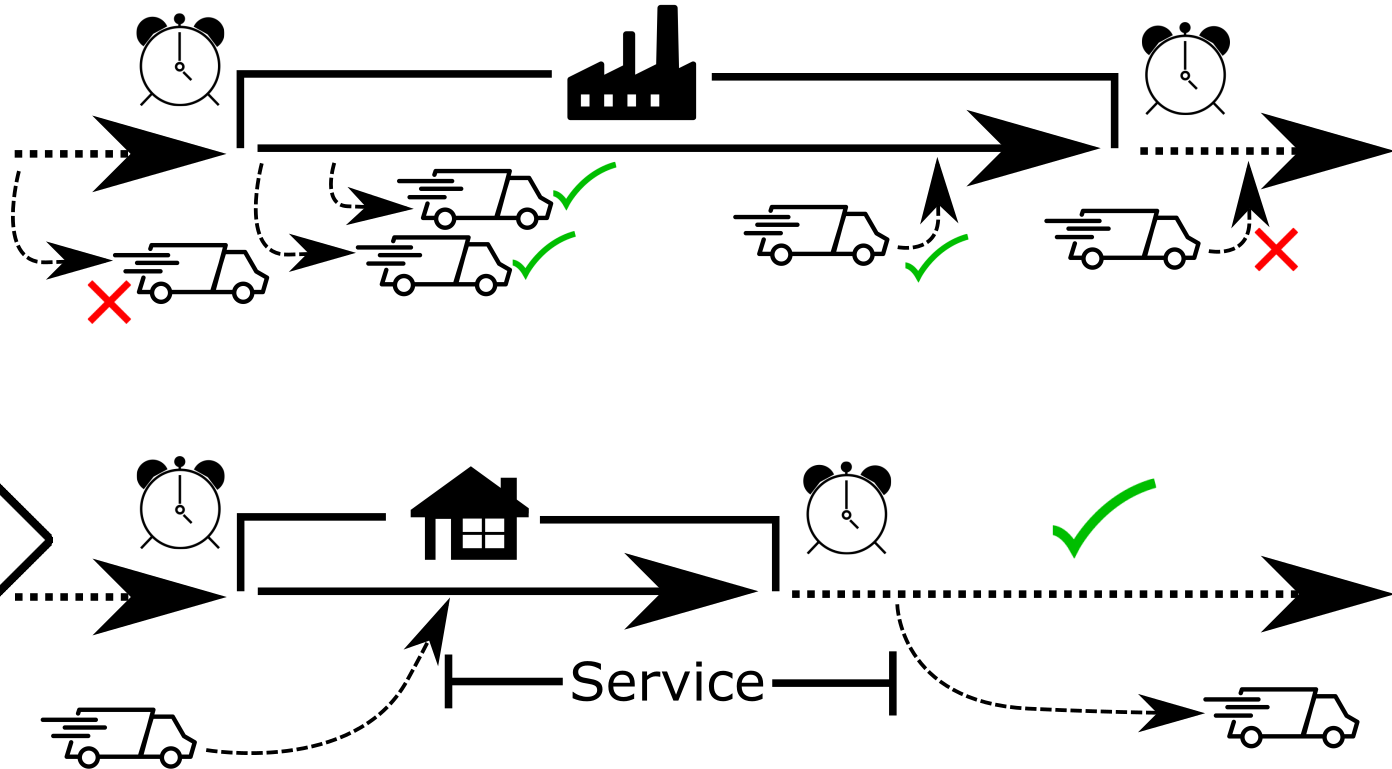
Central Depot
Vehicle Fleet
Customers
Demand
Capacity
Time Windows



CVRPTW

Central Depot
Vehicle Fleet
Customers
Demand
Capacity

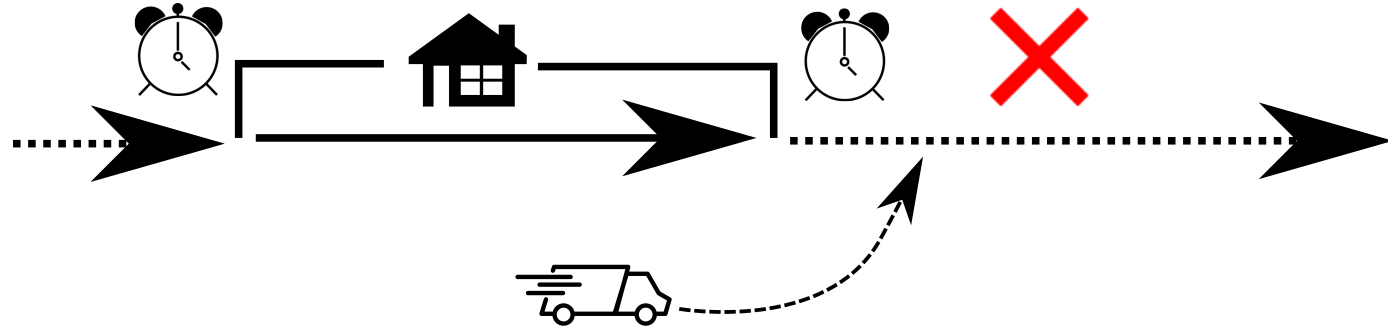
Time Windows



CVRPTW

Central Depot
Vehicle Fleet
Customers
Demand
Capacity

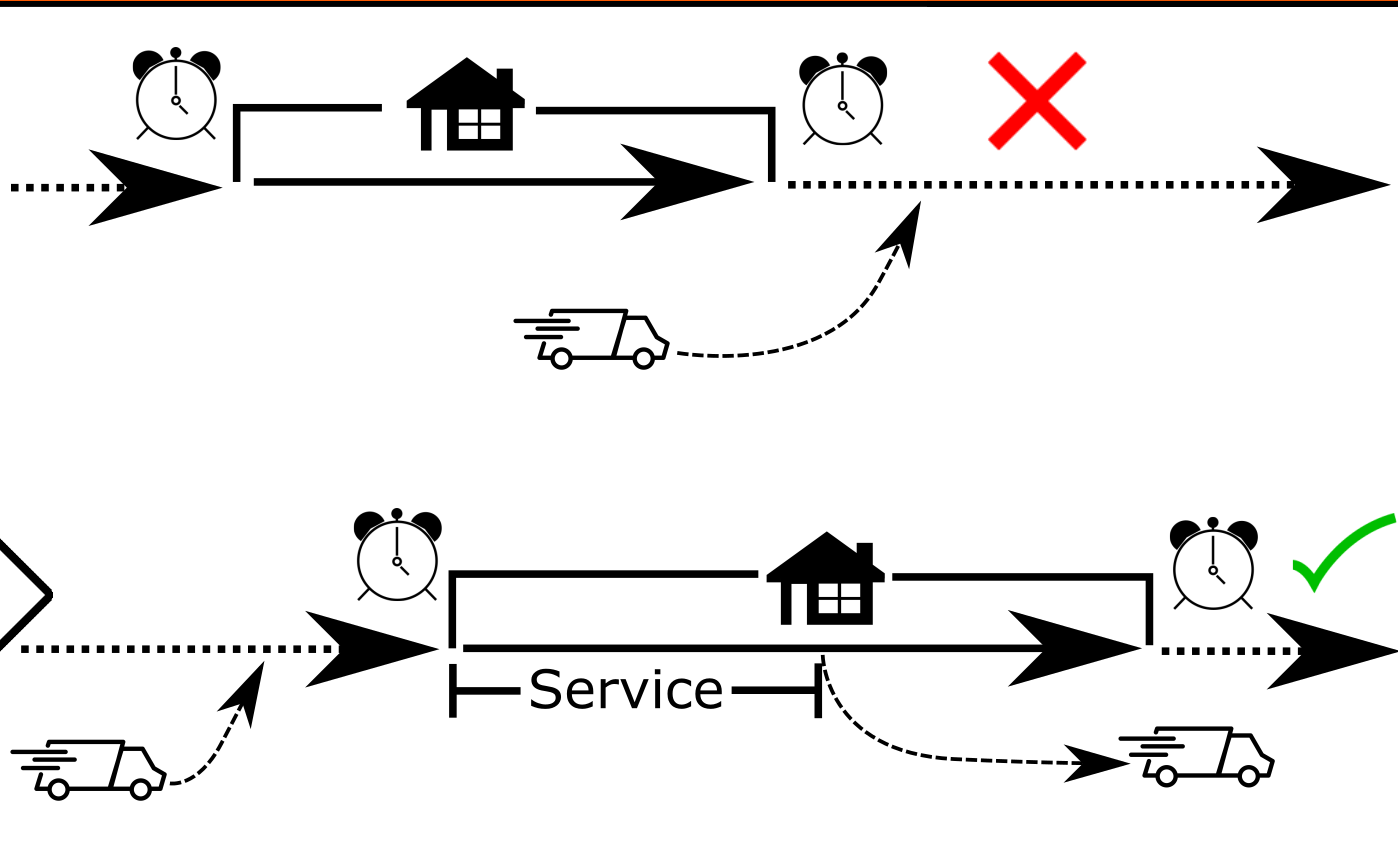
Time Windows



CVRPTW

Central Depot
Vehicle Fleet
Customers
Demand
Capacity

Time Windows



CVRPTW

Central Depot
Vehicle Fleet
Customers
Demand
Capacity
Time Windows
Objective

Minimise the total distance
travelled by all vehicles combined.



Constraint

Programming

Decision Variables

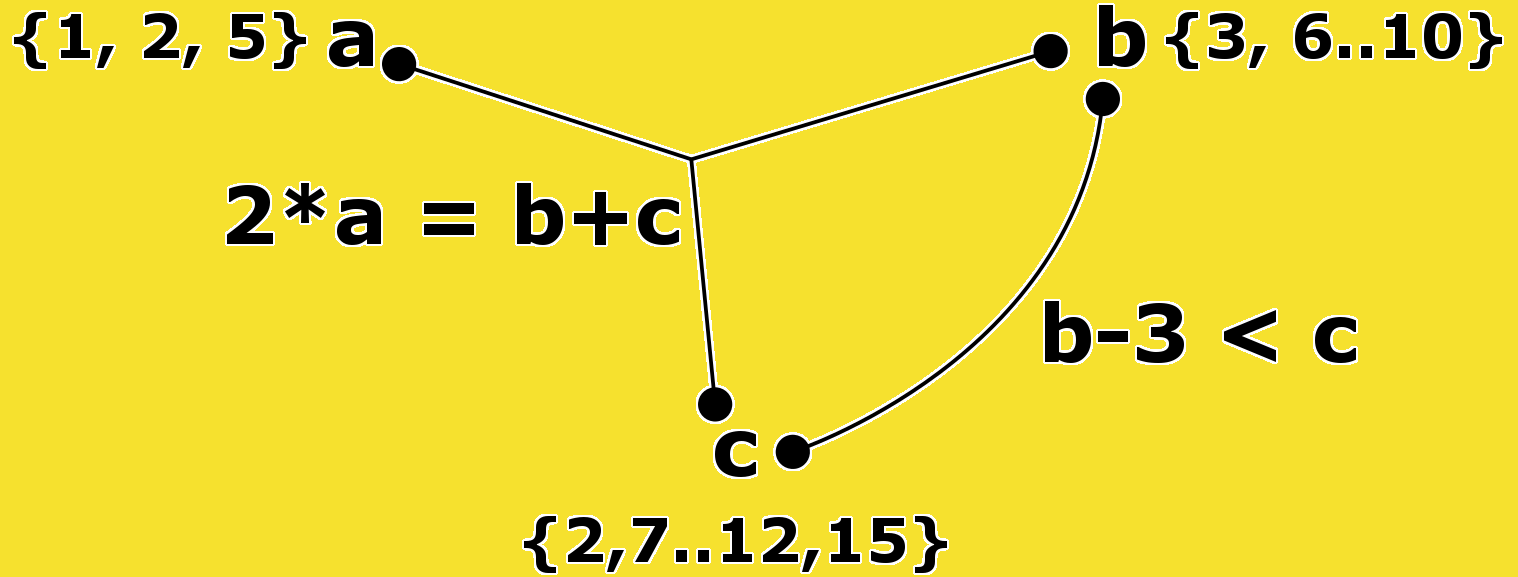
{1, 2, 5} a

b {3, 6..10}

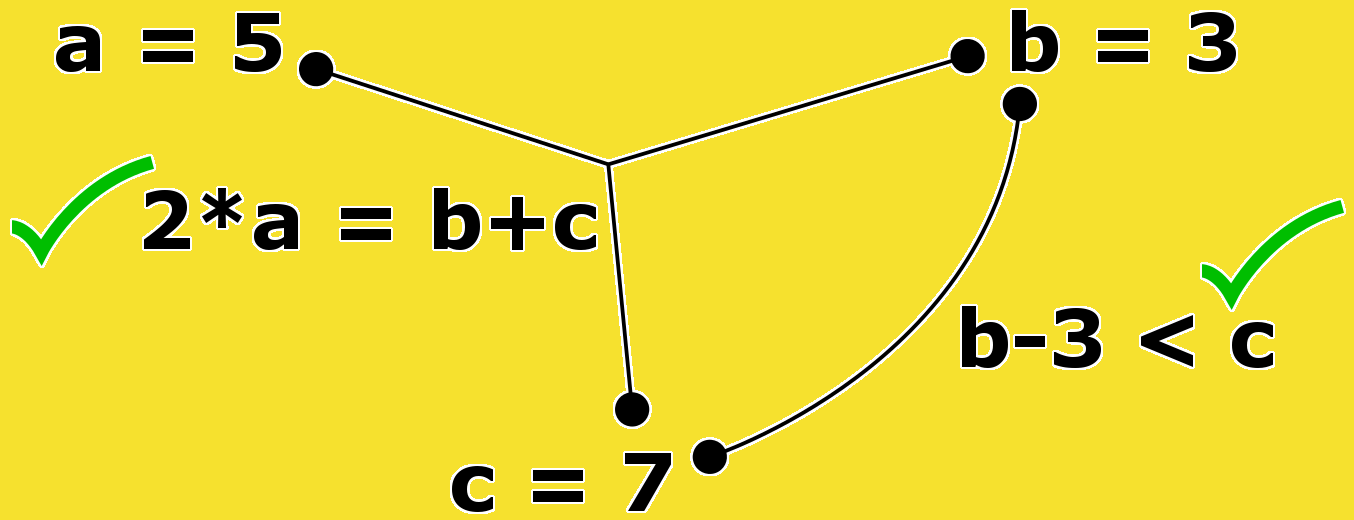
c

{2,7..12,15}

Constraints

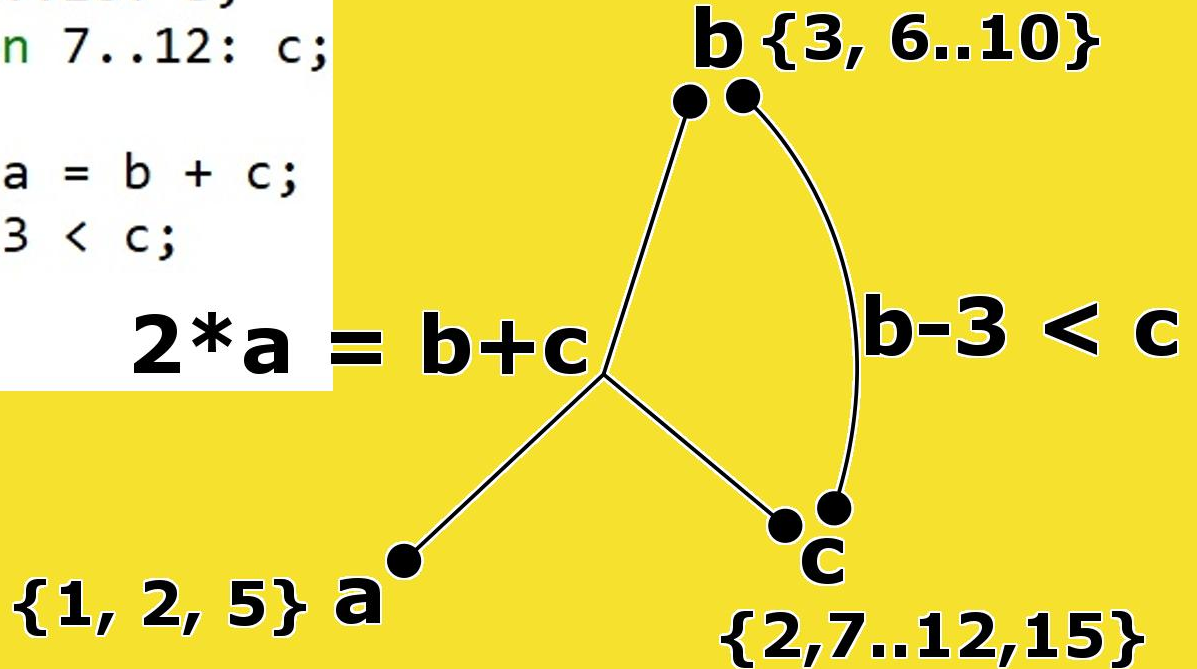


Valid Solution



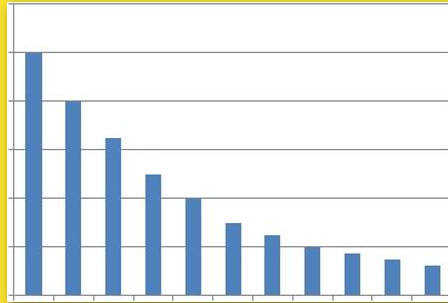
MiniZinc - Source Code

```
1 var {1,2,5}: a;  
2 var {3} union 6..10: b;  
3 var {2,15} union 7..12: c;  
4  
5 constraint 2 * a = b + c;  
6 constraint b - 3 < c;  
7  
8 solve satisfy;
```



Optimisation

```
1 solve minimize total_distance;
```

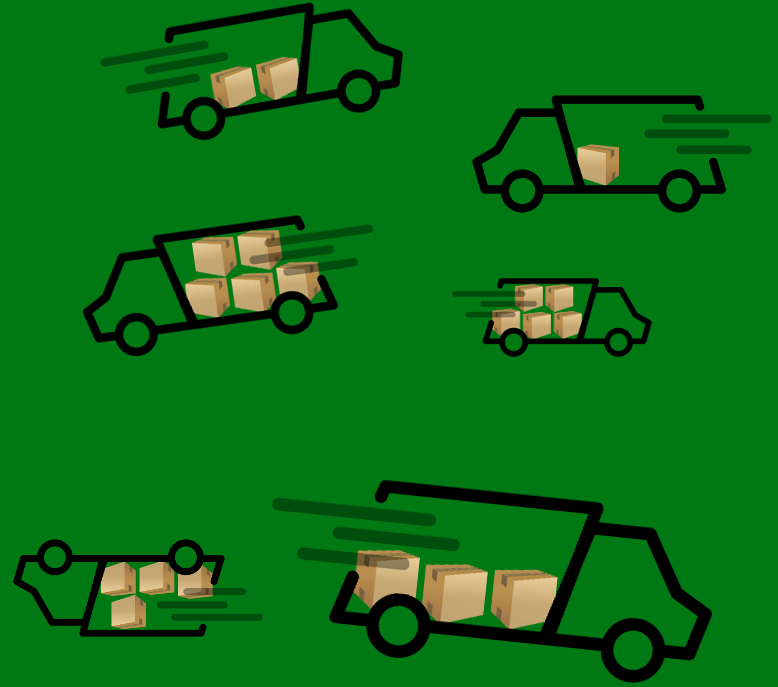
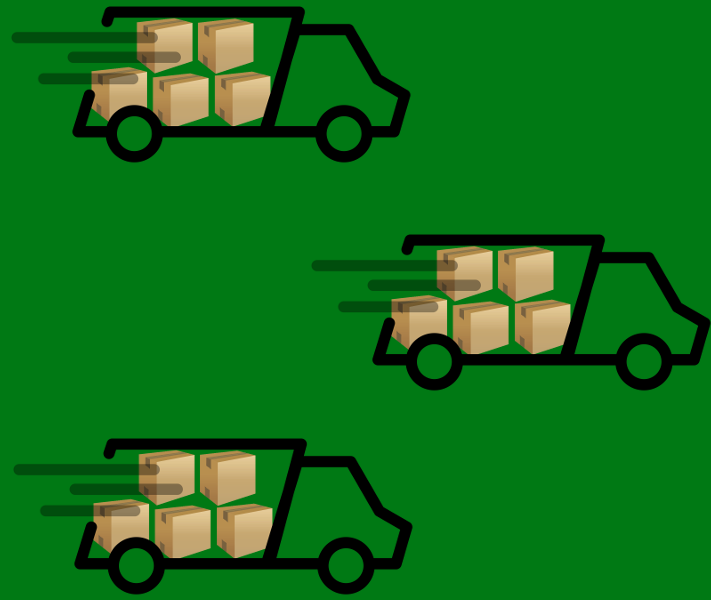


MODEL 2

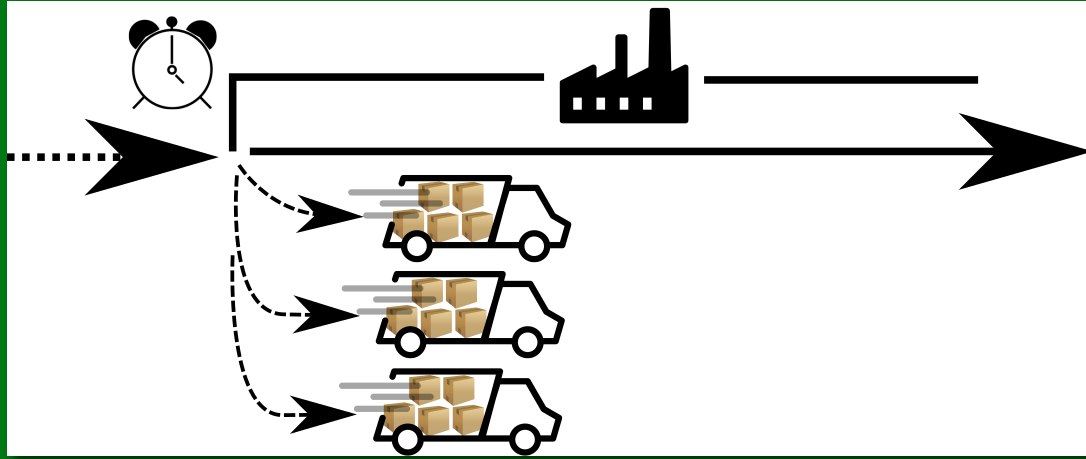
Homogeneous Fleet

VS

Heterogeneous Fleet



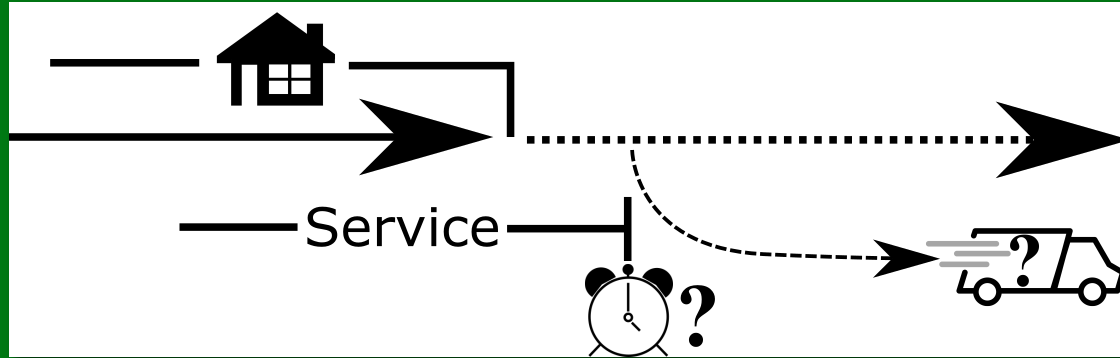
All used vehicles leave the depot



- as early as possible

- fully loaded

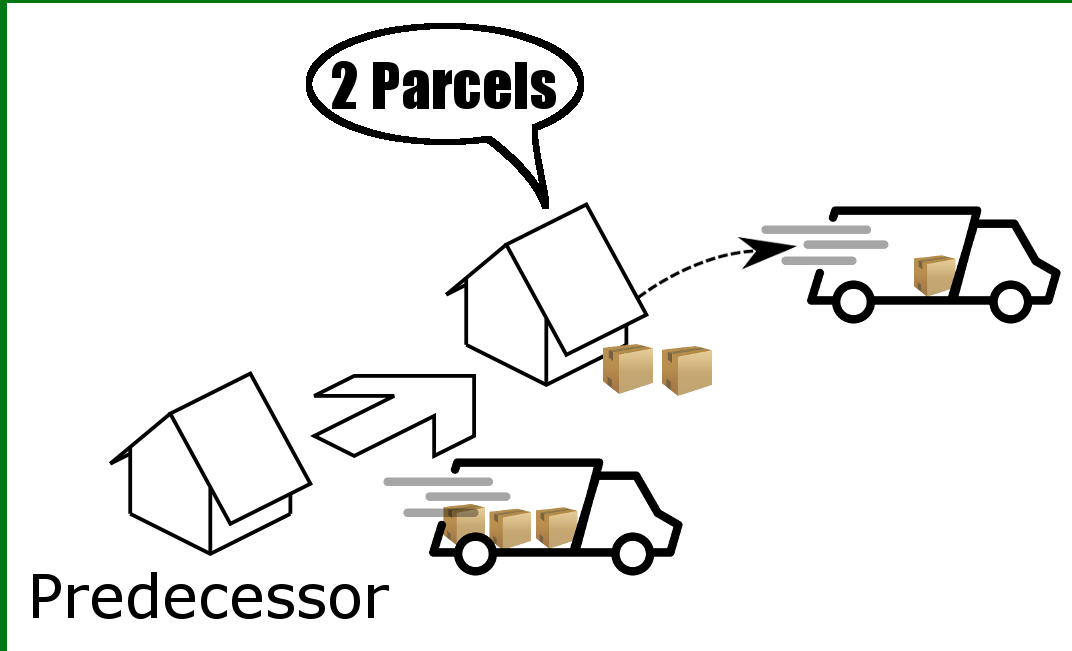
What is the vehicle's



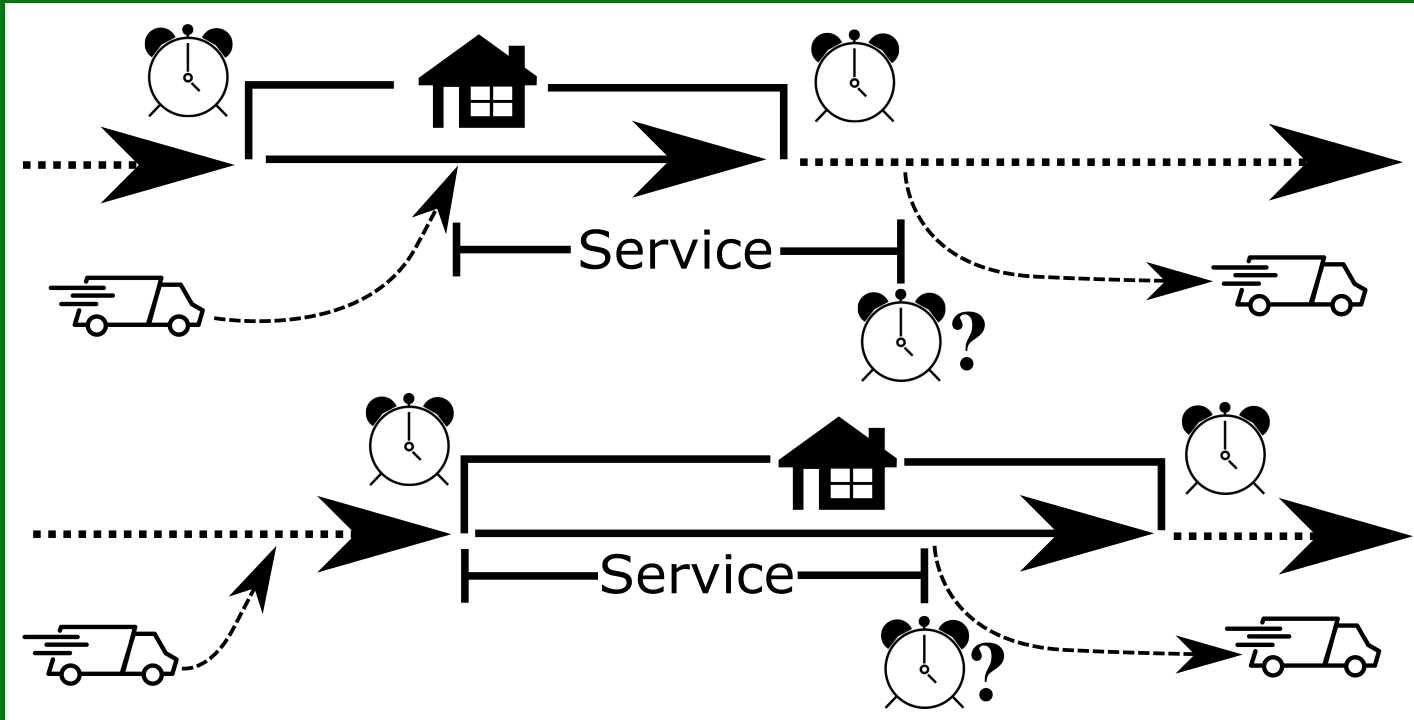
● leaving time

● remaining load

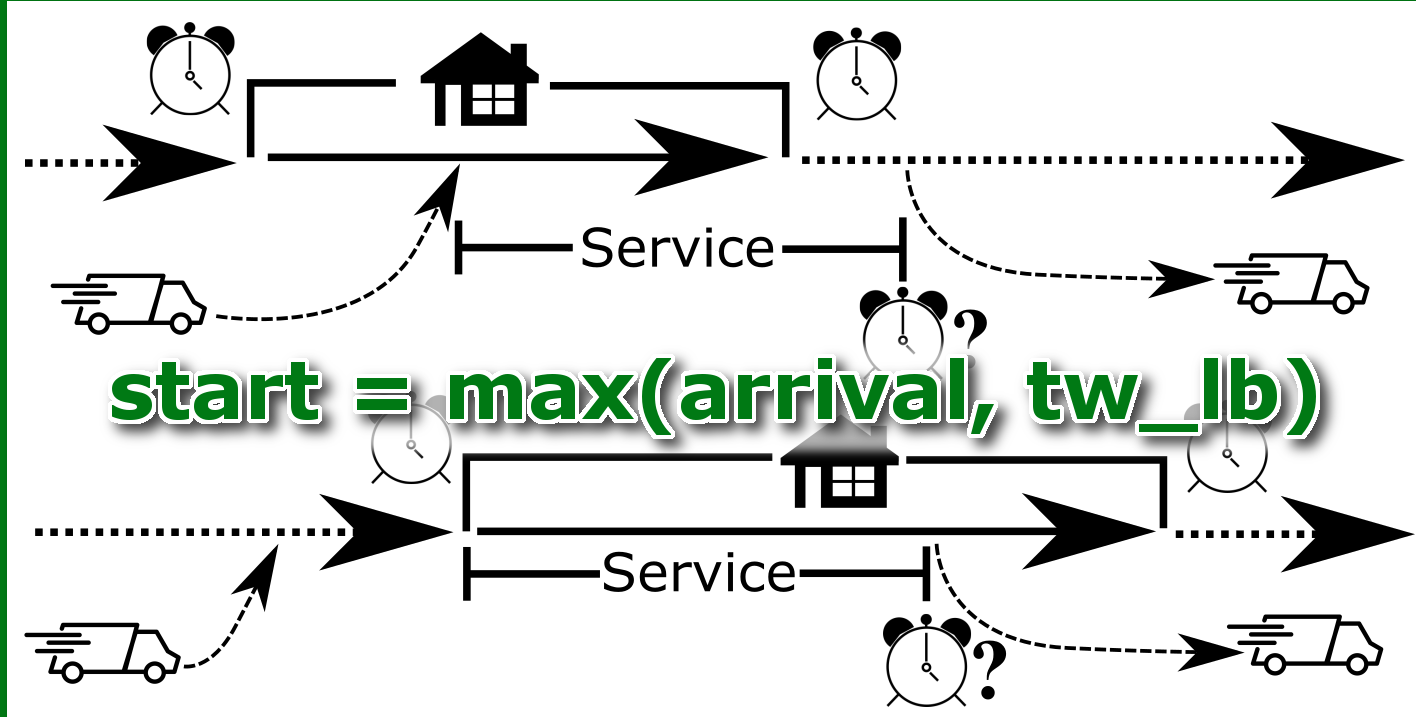
Load Propagation



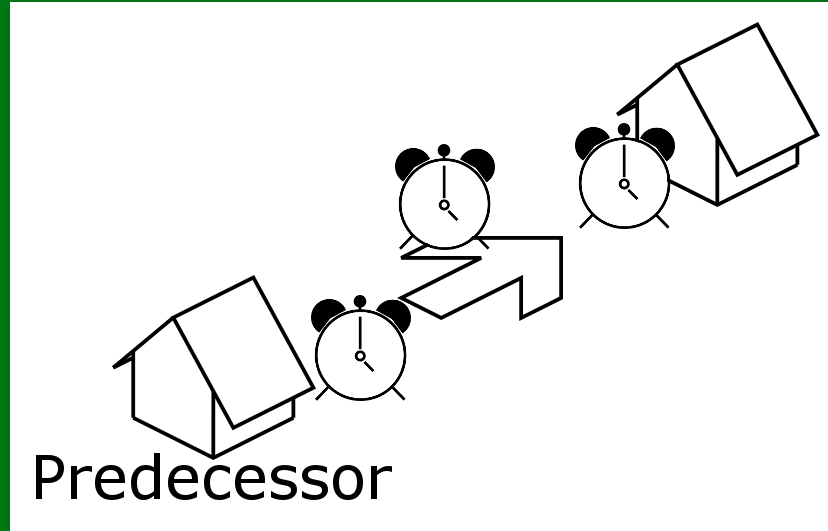
Service Start Time



Service Start Time

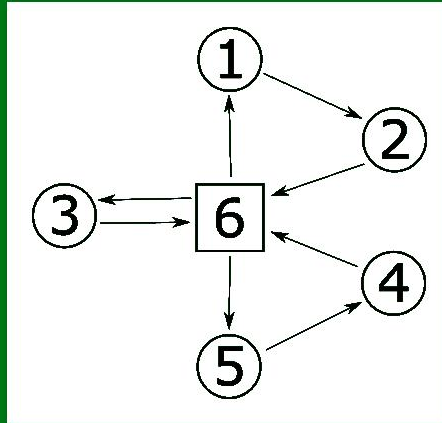


Arrival Time



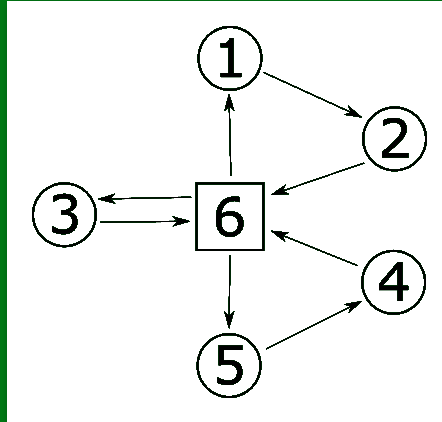
$$\text{arrival} = \text{leave}(\text{pred}) + \text{distance}$$

Predecessor Array



Customer	1	2	3	4	5
<i>pred</i>	6 → 1		6	5 ← 6	6

Predecessor Array



Customer	1	2	3	4	5
<i>pred</i>	6	1	6	5	6
<i>last</i>	0	1	1	1	0

Objective

```
1 solve minimize sum(c in CUSTOMER)
2   (
3     dist[pred[c],c]
4     +
5     dist[c, DEPOT] * last[c]
6   );
```

outcome / 'aʊtkʌm/

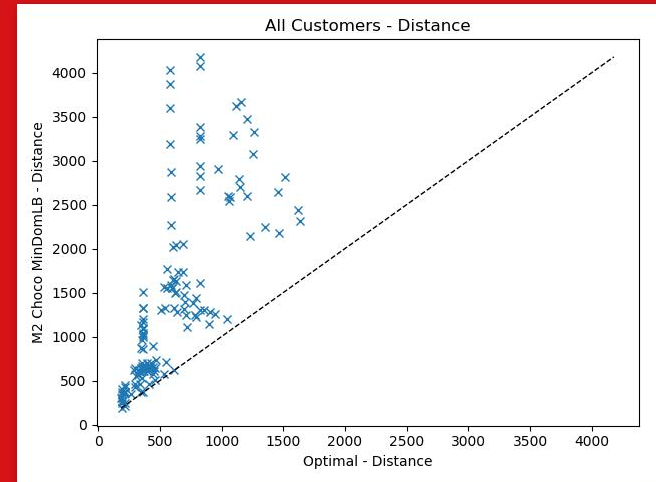
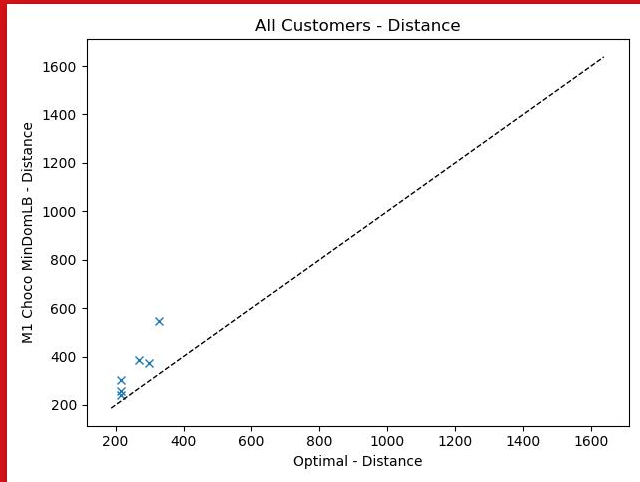
The way a thing turns out

Oxford Dictionary

Model 1

VS

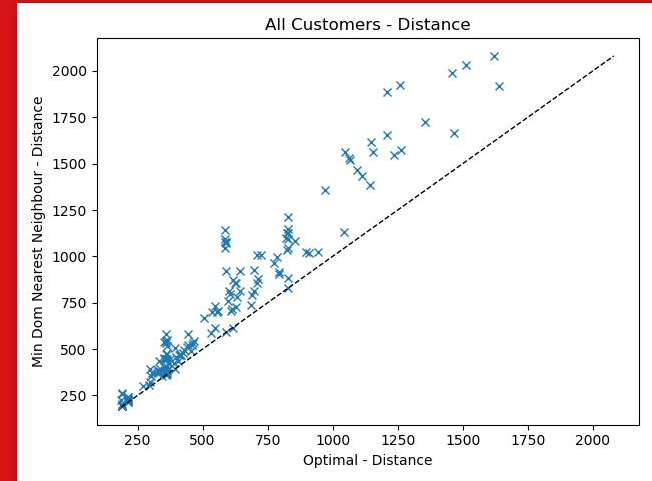
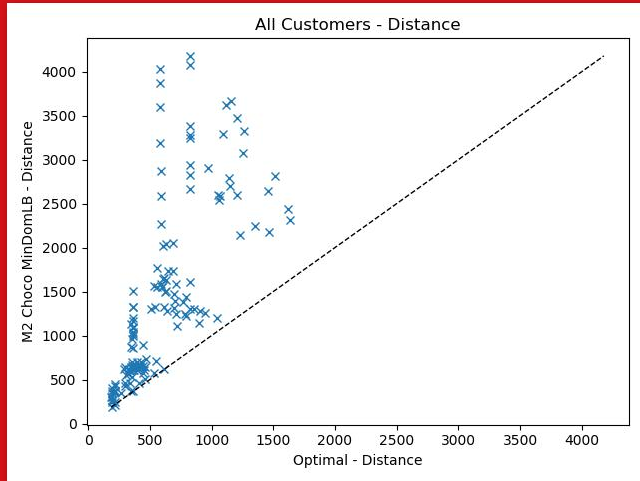
Model 2



Smallest Value

VS

Nearest Neighbour



Thank you!

Questions?



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